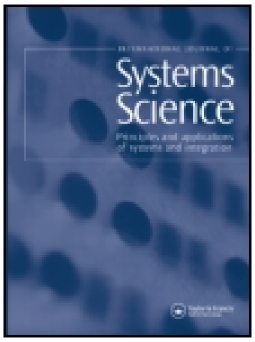

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H_∞ filtering for discrete-time systems subject to stochastic missing measurements: a decomposition approach

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This paper deals with the problem of H_∞ filtering for discrete-time systems with stochastic missing measurements. A new missing measurement model is developed by decomposing the interval of the missing rate into several segments. The probability of the missing rate in each subsegment is governed by its corresponding random variables. We aim to design a linear full-order filter such that the estimation error converges to zero exponentially in the mean square with a less conservatism while the disturbance rejection attenuation is constrained to a given level by means of an H_∞ performance index. Based on Lyapunov theory, the reliable filter parameters are characterised in terms of the feasibility of a set of linear matrix inequalities. Finally, a numerical example is provided to demonstrate the effectiveness and applicability of the proposed design approach.

Keywords: stochastic missing measurement; H_∞ filtering; linear matrix inequalities (LMIs)

1. Introduction

Estimation of dynamic systems has found many practical applications and has attracted a lot of attention during the last decades, see for example, Xu, Lam, and Mao (2007), Zhang and Boukas (2009), Sarkka (2007), Zhang and Han (2006), Shi, Mahmoud, Nguang, and Ismail (2006), Duan, Zhang, Zhang, and Mosca (2006), Zhou, Xu, Chen, and Chu (2011), Yang, Xia, Qiu, and Zhang (2010), You, Gao, and Basin (2013) and the references therein. For instance, filter design aiming to reduce the conservatism aroused from time delay for Markovian jump linear systems with norm-bounded parameter uncertainty and time-varying delay has been investigated by introducing some slack matrix variables in Xu et al. (2007), Zhang and Boukas (2009) and Shi et al. (2006). Unscented Kalman filter to continuous-time filtering problems is addressed in Sarkka (2007). However, these inferences are based on an ideal assumption that the measurement outputs are precise, i.e. the output of measuring instruments has no any deviation or missing measurement.

Measurement inaccuracy may deteriorate the performance of the system and even destabilise the system. However, in practical applications, it is difficult for most of the measuring meters to achieve measurement results precisely due to the systemic error, stochastic error, etc. in the process of the measurement. Although a better result can be attained by using a more precise and expansive instrument, from the cost saving point of view, using a better design method in theory is good choice for customers as long as a good control performance can be obtained. On the other hand, achieving a precise measurement is impossible due to the complicated situation and technical reasons in some cases. For example, in target tracking control problems, the measurement inaccuracy increases with the tracking error and the bad measurement situation.

The system subject to missing measurement has recently received increasing interests due to their extensive application, see Basin, Shi, and Calderon-Alvarez (2010), Nahi (1969), Wang, Yang, Ho, and Liu (2006), Moayedi, Foo, and Soh (2010), Wang, Ho, Liu, and Liu (2009), Wei, Wang, and Shu (2009), Dong, Wang, Ho, and Gao (2010), Gao, Zhao, Lam, and Chen (2009), Gu, Wang, and Yue (2011), Yang and Ye (2007), and You and Yin (2013). For example, Nahi (1969) first developed an optimal recursive filter for systems with missing measurements and the filter is derived via solving two Riccati equations. In Moayedi et al. (2010), an adaptive filtering scheme is proposed for state estimation in sensor networks and/or networked control systems with mixed uncertainties of random measurement delays, packet dropouts and missing measurements.

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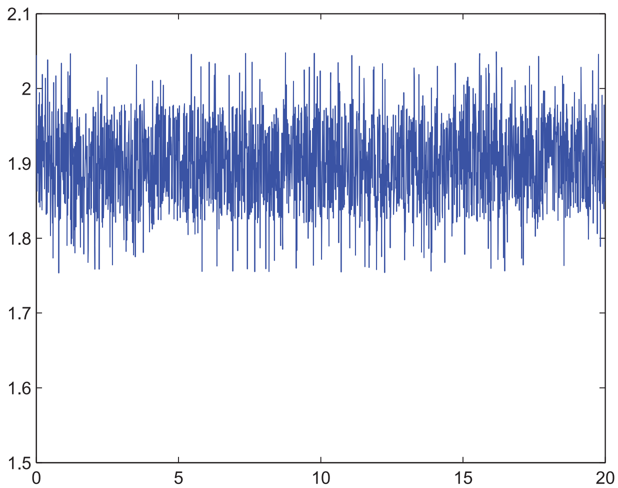

Figure 1. The measurement output.

In Wang et al. (2006) and Gao et al. (2009), the authors developed the missing measurement model as

$$y_k = r_k C x_k \tag{1}$$

where x_k and y_k are the state vector and the measured output vector, respectively, and r_k is a random variable taking the value of 0 or 1, which denotes the output signal is missed when $r_k = 0$. In Wei et al. (2009), Dong et al. (2010) and Gu et al. (2011), r_k in Equation (1) is defined as a diagonal random matrix. Each element of the matrix takes value in interval [0, 1] to characterise the stochastic missing measurement in each sensor channel.

As shown in Figure 1, a measurement output deviates from the real value in a random way. In this case, if the real value of y_k is assumed to be 2.000, the missing rate r_k belongs to [0.875 1.025]. We can obtain the probabilities of the missing rate in subintervals [0.875 0.905), [0.905 0.935), [0.935 0.995) and [0.995 1.025] as 7%, 38%, 46%, 5% and 4%, respectively, by using statistic way. The difference of the probability distribution among those intervals is remarkable (shown in Figure 2). Apparently, it will lead to some conservatism if we still regard the distribution of the missing rate obeys only one statistical features. This motivates us with the present study.

In this paper, an H_∞ filtering design is addressed for the system subject to probabilistic missing measurements. A novel missing measurement model is developed by utilising the statistical feature of the missing rate in every subintervals. By using Lyapunov function approach, sufficient conditions on the H_∞ performance analysis are given and the desired filter parameters related to the statistical feature of the missing rate are also achieved. Simulation results demonstrate the effectiveness of the proposed filter design scheme.

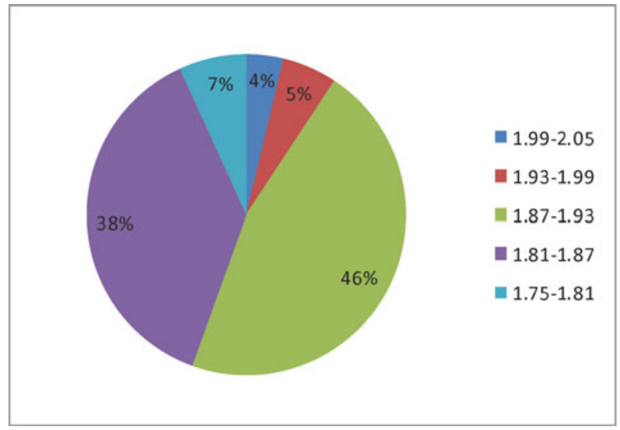

Figure 2. The distribution of the measured output.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices; I is the identity matrix of appropriate dimensions; $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate; the notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite); the notation $X \preceq Y$ for $X, Y \in \mathbb{R}^{m\times m}$ means that every element of the diagonal matrix X is no more than the corresponding one of the diagonal matrix Y. When x is a stochastic variable, $\mathbb{E}\{x\}$ stands for the expectation of x; the asterisk $*$ in a matrix is used to denote term that is induced by symmetry.

2. Problem formulation and preliminaries

Consider the following discrete-time linear system with state time delay,

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-d) + B\omega(k) \\ y(k) = \Pi(k)Cx(k) + D\omega(k) \\ z(k) = Lx(k) \\ z(k) = \phi(k), \quad k = -d, -d+1, \dots, 0 \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $y(k) \in \mathbb{R}^m$ is the measured output vector; $z(k) \in \mathbb{R}^q$ is the signal to be estimated; $\phi(k)$ is the initial condition of the state; $d > 0$ is the time delay; $\omega(k)$ is the deterministic disturbance signal in $l_2[0, \infty)$; $\Pi(k)$ $(0 \preceq \underline{\Pi} \preceq \Pi(k) \preceq \bar{\Pi})$ is a diagonal matrix, which denotes the missing rate matrix; A, A_d, B, C, D and L are system matrices with appropriate dimensions.

As mentioned in Section 1, the missing rate of the measurement output, in general, does not obey a single distribution in the overall interval of the missing measurement. Here, we decompose the interval of the missing rate into several subintervals according to the statistic feature of the missing rate. For convenience of analysis, we define a set $\Omega_i = \{k | \underline{\Pi}_i \preceq \Pi(k) \preceq \bar{\Pi}_i, i \in \mathcal{I} \triangleq \{i | i = 1, 2, \dots, p\}\}$, where $\bar{\Pi}_i = \underline{\Pi}_{i+1}$ and $\underline{\Pi}_1 = \underline{\Pi}$, $\bar{\Pi}_p = \bar{\Pi}$, from which we can know that $\cup_{i=1}^{p}\Omega_i = \mathbb{R}^+$ and $\Omega_i \cap \Omega_j$ is an empty set for $i, j \in \mathcal{I}, i \neq j$. Therefore, the missing rate takes value in the ith interval when $k \in \Omega_i$. It should be noted that which subinterval k belongs to is a random event; therefore, we define a stochastic variable $\alpha_i(k)$ as

$$\alpha_i(k) = \begin{cases} 1 & k \in \Omega_i \\ 0 & \text{others} \end{cases}$$

Based on the above discussion, the measured output is then modelled by

$$\tilde{y}(k) = \sum_{i=1}^{p} \alpha_i(k)\Pi_i(k)Cx(k) + D\omega(k) \tag{3}$$

where $\Pi_i(k) = \text{diag}\{\pi_{i1}(k), \pi_{i2}(k), \ldots, \pi_{im}(k)\}(i \in \mathcal{I})$, π_{ij} denotes the missing rate in each measurement channel. We define $\text{Prob}\{\alpha_i(k) = 1\} = \bar{\alpha}_i$ and $\sum_{i=1}^{p} \bar{\alpha}_i = 1$.

Remark 1: If $\alpha_i(k) \equiv 1$ and $\Pi_i(k) \equiv 1$, then the problem reduces to a normal filter design.

Remark 2: If one selects $p = 1$, $\text{Prob}\{\alpha_1(k) = 1\}$ becomes 100%, then it reduces to the case of the missing rate varying in a single interval.

Remark 3: In model (3), we assume the expectation of the random variable $\alpha_i(k)$ and its variance σ_i are known in prior. The more distribution information of the missing rate we know, the larger number p can be set, and the interval endpoint depends on the prior selection of the statistic feature. It should be noted that a less conservative result can be attained with the increasing number p, which will be illustrated in Section 5. It is a trade-off between the performance and the difficulty of the acquisition of the statistic feature.

For analysis convenience, here, we define $\Pi_{i0} = \frac{1}{2}(\bar{\Pi}_i + \underline{\Pi}_i)$ and $\Pi_{i1} = \frac{1}{2}(\bar{\Pi}_i - \underline{\Pi}_i)$. Then $\Pi_i(k)$ can be rewritten as

$$\Pi_i(k) = \Pi_{i0} + \Delta_i(k)\Pi_{i1} \tag{4}$$

where $\Delta_i(k)$ is a unknown matrix function satisfying

$$\Delta_i^T(k)\Delta_i(k) \leq I \tag{5}$$

In this paper, we are interested in designing a linear filter for the estimation of $z(k)$ in Equation (2),

$$\begin{cases} x_f(k+1) = A_f x_f(k) + B_f \tilde{y}(k) \\ z_f(k) = L_f x_f(k) \end{cases} \tag{6}$$

where $x_f(k) \in \mathbb{R}^n$ and $z_f(k) \in \mathbb{R}^q$; A_f, B_f and C_f are matrices to be determined.

Combining Equations (2) and (6), the filtering error dynamics can be represented as

$$\begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \tilde{A}\bar{x}(k) + \bar{A}_d\bar{x}(k-d) + \bar{B}\omega(k) \\ \bar{e}(k) = \bar{L}\bar{x}(k) \end{cases} \tag{7}$$

where $\bar{x}(k) = \left[x^T(k)\; x_f^T(k)\right]^T$, $e(k) = z(k) - z_f(k)$ and

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_f \sum_{i=1}^{p} \bar{\alpha}_i \Pi_i(k) C & A_f \end{bmatrix},$$
$$\tilde{A} = \sum_{i=1}^{p}(\alpha_i(k) - \bar{\alpha}_i)\begin{bmatrix} 0 & 0 \\ B_f \Pi_i(k) C & 0 \end{bmatrix}$$
$$\bar{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ B_f D \end{bmatrix}, \bar{L} = [L - L_f].$$

In this paper, we aim to design the filter gain matrices in Equation (6), such that the following requirements are simultaneously satisfied:

- the zero-solution of the augmented system (7) with $\omega(k) = 0$ is asymptotically stable in the mean square;
- under the zero-initial condition, the filtering error $e(k)$ satisfies

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \|e(k)\|^2\right\} \leq \mathbb{E}\left\{\sum_{k=0}^{\infty} \gamma^2 \|\omega(k)\|^2\right\} \tag{8}$$

for all nonzero $\omega(k)$, where $\gamma > 0$ is a given disturbance attenuation level.

The following lemma is useful in deriving the criteria.

Lemma 2.1 (Boyd, El-Ghaoui, Feron, Balakrishnan, & Yaz, 1997)**:** *Let $\mathcal{F} = \mathcal{F}^T$, $\mathcal{M}$ and $\mathcal{N}$ be real matrices of appropriate dimensions with Δ satisfying $\Delta^T\Delta \leq I$. Then,*

$$\mathcal{F} + \mathcal{M}\Delta\mathcal{N} + \mathcal{N}^T\Delta^T\mathcal{M}^T \leq 0$$

if and only if there exists a positive scalar $\varepsilon > 0$ such that

$$\mathcal{F} + \varepsilon\mathcal{M}\mathcal{M}^T + \varepsilon^{-1}\mathcal{N}^T\mathcal{N} \leq 0$$

or equivalently

$$\begin{bmatrix} \mathcal{F} & * & * \\ \mathcal{M}^T & -\varepsilon I & * \\ \varepsilon\mathcal{N} & 0 & -\varepsilon I \end{bmatrix} \leq 0$$

3. Stability and H_∞ performance analysis

At first, we establish criteria of mean-square stability and H_∞ performance for the filtering error dynamics (7), which plays a fundamental role in the derivation of our H_∞ filter design method.

Theorem 3.1: *Consider the system* (2) *subject to probabilistic missing measurements. Given a scalar* $\gamma > 0$ *and the filter parameters* A_f, B_f *and* C_f. *If there exist positive-definite matrices* $P = P^T > 0$, $Q = Q^T > 0$ *satisfying Equation* (9)*, then the filtering error system* (7) *is mean-square stable with* H_∞ *filtering performance,*

$$\Phi_1 = \begin{bmatrix} \Gamma_1 & * & * & * \\ \tilde{L} & -I & * & * \\ P\mathcal{A} & 0 & -P & * \\ P\tilde{\mathcal{A}} & 0 & 0 & -P \end{bmatrix} < 0 \tag{9}$$

where $\Gamma_1 = diag\{-P + Q, -Q, -\gamma^2 I\}$, $\mathcal{A} = [\bar{A}\ \bar{A}_d\ \bar{B}]$, $\tilde{L} = [\bar{L}\ 0\ 0]$,

$$\tilde{\mathcal{A}} = \left[\sum_{i=1}^{p} \sigma_i \begin{bmatrix} 0 & 0 \\ B_f \Pi_i(k) C & 0 \end{bmatrix}, 0, 0\right].$$

Proof: Define a Lyapunov functional candidate as

$$V(k) = \bar{x}^T(k) P \bar{x}(k) + \sum_{i=k-d}^{k-1} \bar{x}^T(i) Q \bar{x}(i) \tag{10}$$

Calculating the difference of $V(k)$ along the system (7) and taking its mathematical expectation, we have

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\left\{\bar{x}^T(k+1) P \bar{x}(k+1)\right\} \\ &\quad + \mathbb{E}\left\{\bar{x}^T(k)(Q - P)\bar{x}(k)\right\} \\ &\quad - \mathbb{E}\left\{\bar{x}^T(k-d) Q \bar{x}(k-d)\right\} \end{aligned} \tag{11}$$

Note that $\mathbb{E}\{\alpha_i(k) - \bar{\alpha}_i\} = 0$ and

$$\mathbb{E}\{(\alpha_i(k) - \bar{\alpha}_i)(\alpha_j(k) - \bar{\alpha}_j)\} = \begin{cases} \sigma_i^2 & i = j \\ 0 & i \neq j \end{cases} \qquad i, j \in \mathcal{I}$$

Then we have

$$\begin{aligned} &\mathbb{E}\{\bar{x}^T(k+1) P \bar{x}(k+1)\} \\ &\quad = \mathbb{E}\left\{\xi^T(k)\left[\mathcal{A}^T P \mathcal{A} + \tilde{\mathcal{A}}^T P \tilde{\mathcal{A}}\right]\xi(k)\right\} \end{aligned} \tag{12}$$

where $\xi(k) = [\bar{x}^T(k)\ \bar{x}^T(k-d)\ \omega^T(k)]^T$.

Putting Equation (12) into Equation (11), we obtain

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\left\{\xi^T(k)(\mathcal{A}^T P \mathcal{A} + \tilde{\mathcal{A}}^T P \tilde{\mathcal{A}})\xi(k)\right\} \\ &\quad + \mathbb{E}\{\bar{x}^T(k)(Q - P)\bar{x}(k) \\ &\quad - \bar{x}^T(k-d) Q \bar{x}(k-d)\} \end{aligned}$$

Using Schur complement for Equation (9) and recalling the definition of $\bar{e}(k)$ in Equation (7), we have

$$\mathbb{E}\{\Delta V(k) + \bar{e}^T(k)\bar{e}(k) - \gamma^2 \omega^T(k)\omega(k)\} \leq 0 \tag{13}$$

If we choose $\omega(k) = 0$, one can conclude that $\mathbb{E}\{\Delta V(k)\} \leq -\epsilon \|\bar{x}(k)\|^2$ for a sufficiently small $\epsilon > 0$ and $x(k) \neq 0$, and thus the mean square stability for the system (7) with $\omega(k) = 0$ is established.

Next, when $\omega(k) \neq 0$, we can obtain

$$\begin{aligned} \mathbb{E}\left\{\sum_{k=0}^{\infty} e^T(k) e(k)\right\} &\leq \mathbb{E}\left\{\sum_{k=0}^{\infty} \gamma^2 \omega^T(k)\omega(k)\right\} \\ &\quad + \mathbb{E}\{V(0) - V(\infty)\} \end{aligned}$$

by summing up Equation (13) from 0 to ∞ with respect to k on both sides of Equation (13). Under zero conditions, it is straightforward to see that

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} e^T(k) e(k)\right\} \leq \mathbb{E}\left\{\sum_{k=0}^{\infty} \gamma^2 \omega^T(k)\omega(k)\right\}$$

Recalling the requirement of the filter design in Section 2, the proof of Theorem 3.1 is then completed. □

Theorem 3.2: *Consider the system* (2) *subject to probabilistic missing measurements. Given a scalar* $\gamma > 0$, *the statistic feature parameters related to the missing measurements* $\bar{\alpha}_i$, $\underline{\Pi}_i$ *and* $\bar{\Pi}_i$, *and the filter parameters* A_f, B_f *and* C_f. *If there exist positive-definite matrices* $P = P^T > 0$, $Q = Q^T > 0$ *and scalars* ε_i $(i \in \mathcal{I})$ *satisfying Equation* (14)*, then the filtering error system* (7) *is mean-square stable with* H_∞ *filtering performance,*

$$\Xi = \begin{bmatrix} \bar{\Phi}_1 & * & * \\ \Phi_2 & \Phi_3 & * \\ \Phi_4 & 0 & \bar{\Phi}_3 \end{bmatrix} < 0 \tag{14}$$

where

$$\bar{\Phi}_1 = \begin{bmatrix} \Gamma_1 & * & * & * \\ \tilde{L} & -I & * & * \\ P\mathcal{A}_0 & 0 & -P & * \\ P\tilde{\mathcal{A}}_0 & 0 & 0 & -P \end{bmatrix},$$

$$\Phi_2 = \left[\Phi_{21}^T \ \cdots \ \Phi_{2p}^T\right]^T, \Phi_{2i} = \begin{bmatrix} 0 & 0 & \Gamma_{2i}^T & 0 \\ 0 & 0 & 0 & \Gamma_{3i}^T, \end{bmatrix},$$

$$\Gamma_{2i} = P\begin{bmatrix} 0 \\ \bar{\alpha}_i B_f \Pi_{i1} \end{bmatrix}, \Gamma_{3i} = P\begin{bmatrix} 0 \\ \sigma_i B_f \Pi_{i1} \end{bmatrix},$$

$$\Phi_3 = -\mathrm{diag}\{\varepsilon_1 I^{2n\times 2n}, \ldots, \varepsilon_p I^{2n\times 2n}\},$$

$$\bar{\Phi}_3 = -\mathrm{diag}\{\varepsilon_1 I^{n\times n}, \ldots, \varepsilon_p I^{n\times n}\},$$

$$\Phi_4 = \left[\varepsilon_1 \Phi_{41} \ldots \varepsilon_p \Phi_{4p}\right], \Phi_{4i} = [[C\ 0]\ 0\ 0\ 0\ 0 \ldots 0],$$

$$\mathcal{A}_0 = \left[\bar{A}_0\ \bar{A}_d\ \bar{B}\right], \bar{A}_0 = \begin{bmatrix} A & 0 \\ \sum_{i=1}^{p} \bar{\alpha}_i B_f \Pi_{i0} C & A_f \end{bmatrix},$$

$$\tilde{\mathcal{A}}_0 = \left[\sum_{i=1}^{p} \sigma_i \begin{bmatrix} 0 & 0 \\ B_f \Pi_{i0} C & 0 \end{bmatrix} 0\ 0\right], i \in \mathcal{I}.$$

Proof: Recalling the definition of $\Pi(k)$ in Equation (4), we have

$$\Phi_1 = \bar{\Phi}_1 + \sum_{i=1}^{p} \left\{ \Phi_{2i} \Delta_i(k) \Phi_{4i} + \Phi_{4i}^T \Delta_i(k) \Phi_{2i}^T \right\} \quad (15)$$

By using Lemma 2.1, it yields

$$\Phi_1 \leq \bar{\Phi}_1 + \sum_{i=1}^{p} \left\{ \varepsilon_i \Phi_{2i} \Phi_{2i}^T + \varepsilon_i^{-1} \Phi_{4i}^T \Phi_{4i} \right\} \quad (16)$$

which is equivalent to Equation (14) by using Schur complement. This completes the proof. □

4. H_∞ filtering design

In this section, we provide a solution to the filtering design problem for the systems (2) subject to the stochastic missing measurements. Our immediate goal is to transform inequalities (14) into an explicit linear matrix inequalities (LMI) with the unknown filter matrices.

Theorem 4.1: *Consider the system* (2) *subject to probabilistic missing measurements. Given a scalar $\gamma > 0$ and the parameters related to the missing measurements $\bar{\alpha}_i$, $\underline{\Pi}_i$ and $\bar{\Pi}_i$, there exists a filter in the form of Equation* (6) *such that the filtering error system* (7) *is mean-square stable with H_∞ filtering performance, if there exist positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, and scalars $\varepsilon_i > 0$ $(i \in \mathcal{I})$ such that the following LMI holds:*

$$\bar{\Xi} = \begin{bmatrix} \tilde{\Phi}_1 & * & * \\ \tilde{\Phi}_2 & \Phi_3 & * \\ \tilde{\Phi}_4 & 0 & \tilde{\Phi}_3 \end{bmatrix} < 0 \quad (17)$$

where

$$\tilde{\Phi}_1 = \begin{bmatrix} \tilde{\Gamma}_1 & * & * & * \\ \hat{L} & -I & * & * \\ \Psi_1 & 0 & -\bar{P} & * \\ \Psi_2 & 0 & 0 & -\bar{P} \end{bmatrix}$$

$$\tilde{\Gamma}_1 = \mathrm{diag}\{-\bar{P} + \bar{Q}, -\bar{Q}, -\gamma^2 I\}, \ \bar{P} = \begin{bmatrix} P_1 & R \\ R & R \end{bmatrix}$$

$$\hat{L} = \left[\left[L \ -\bar{L}_f \right] 0 \ 0 \right]$$

$$\Psi_1 = \left[\tilde{A}_0 \ \tilde{A}_d \ \tilde{B} \right], \ \Psi_2 = \left[\varphi \ 0 \ 0 \right]$$

$$\tilde{A}_0 = \begin{bmatrix} P_1 A + \sum_{i=1}^{p} \bar{\alpha}_i \bar{B}_f \Pi_{i0} C & \bar{A}_f \\ RA + \sum_{i=1}^{p} \bar{\alpha}_i \bar{B}_f \Pi_{i0} C & \bar{A}_f \end{bmatrix}, \ \tilde{A}_d = \begin{bmatrix} P_1 A_d & 0 \\ R A_d & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} P_1 B + \bar{B}_f D \\ RB + \bar{B}_f D \end{bmatrix}, \ \varphi = \begin{bmatrix} \sum_{i=1}^{p} \sigma_i \bar{B}_f \Pi_{i0} C & 0 \\ \sum_{i=1}^{p} \sigma_i \bar{B}_f \Pi_{i0} C & 0 \end{bmatrix}$$

$$\tilde{\Phi}_2 = \left[\tilde{\Phi}_{21}^T \cdots \tilde{\Phi}_{2p}^T \right]^T, \ \tilde{\Phi}_{2i} = \begin{bmatrix} 0 & 0 & \tilde{\Gamma}_{2i} & 0 \\ 0 & 0 & 0 & \tilde{\Gamma}_{3i} \end{bmatrix} (i \in \mathcal{I})$$

$$\tilde{\Gamma}_{2i} = \left[\bar{\alpha}_i \Pi_{i1} \bar{B}_f^T \ \bar{\alpha}_i \Pi_{i1} \bar{B}_f^T \right], \ \tilde{\Gamma}_{3i} = \left[\sigma_i \Pi_{i1} \bar{B}_f^T \ \sigma_i \Pi_{i1} \bar{B}_f^T \right]$$

Furthermore, if Equation (17) *is true, the desired filter parameters are given by*

$$A_f = \bar{A}_f R^{-1}, \ B_f = \bar{B}_f, \ L_f = \bar{L}_f R^{-1}$$

Proof: We are about to prove the conclusion using Theorem 3.2. Partition the matrix P as $P = \left[\begin{smallmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{smallmatrix}\right]$, where $P_1 > 0$, $P_3 > 0$ and P_2 is invertible. Define

$$J := \begin{bmatrix} I & 0 \\ 0 & P_2 P_3^{-1} \end{bmatrix}, \ \mathcal{T} := \mathrm{diag}\{J, J, I, I, J, J, \underbrace{I, \ldots, I}_{2p}\},$$

$$\bar{Q} := J Q J^T,$$

$$R := P_2 P_3^{-1} P_2^T, \ \bar{A}_f = P_2 A_f P_3^{-1} P_2^T, \ \bar{B}_f := P_2 B_f, \ \bar{L}_f := L_f P_3^{-1} P_2^T$$

Pre- and post-multiplying Ξ in Equation (14) with $\mathcal{T}$ and its transpose, we have

$$\mathcal{T} \Xi \mathcal{T}^T = \bar{\Xi} \quad (18)$$

where $\bar{\Xi}$ is defined in Equation (17). If Equation (18) holds, i.e. $\bar{\Xi} < 0$, then $\Xi < 0$, which means the filtering system (2) has a prescribed H_∞ performance γ. From the above definition, the filter parameters can be rewritten as

$$A_f = P_2^{-1} \bar{A}_f R^{-1} P_2, \ B_f = P_2^{-1} \bar{B}_f, \ L_f = \bar{L}_f R^{-1} P_2 \quad (19)$$

Since the following systems are algebraically equivalent as in Zhang and Han (2008),

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline L_f & \end{array}\right] = \left[\begin{array}{c|c} P_2^{-1} \bar{A}_f R^{-1} P_2 & P_2^{-1} \bar{B}_f \\ \hline \bar{L}_f R^{-1} P_2 & \end{array}\right] = \left[\begin{array}{c|c} \bar{A}_f R^{-1} & \bar{B}_f \\ \hline \bar{L}_f R^{-1} & \end{array}\right] \quad (20)$$

This completes the proof. □

Remark 4: The system with constant time delay is considered in this paper as our attention mainly focuses on the filter design for the system with stochastic missing measurements, it is easy to extend the presented results to the system with time-varying delay by using the method similar

Table 1. The distribution of the missing rate.

	Missing rate	p	$\gamma_{\min}$
Case 1	$\underline{\Pi}_1 = 0.6, \bar{\Pi}_1 = 0.8, \bar{\alpha}_1 = 0.2$ $\underline{\Pi}_2 = 0.8, \bar{\Pi}_2 = 1.0, \bar{\alpha}_2 = 0.8$	2	3.97
Case 2	$\underline{\Pi}_1 = 0.6, \bar{\Pi}_1 = 1.0, \bar{\alpha}_1 = 1.0$	1	4.47

to He, Wu, She, and Liu (2004), Yue and Han (2005) and Gao and Chen (2007).

5. Examples

In this section, we present a numerical example to illustrate the usefulness of the developed method on the design of H_∞ filter for the discrete system subject to probabilistic missing measurements.

Consider the system (2) with parametric matrices as follows:

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.7 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & 0.01 \\ -0.1 & -0.1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, L = \begin{bmatrix} 1 & 2 \end{bmatrix}, d = 5$$

.

Two cases are listed in Table 1 from which one can know the missing rate varies from 0.6 to 1.0 in a stochastic way. In Case 1, the distribution of the missing rate between the two subintervals, i.e. [0.6 0.8) and [0.8 1.0), is markedly different. The expectations of the missing rate in these two subintervals are 0.2 and 0.8, respectively. However, in Case 2, the distribution does not differentiate between these two subintervals.

By using Theorem 4.1, one can obtain the minimised feasible γ for the filtering problem is $\gamma_{\min} = 3.97$ under the condition of Case 1, while $\gamma_{\min} = 4.47$ under the condition of Case 2, which has illustrated that a less conservative result can be obtained than the one without considering its detailed missing rate distribution. Also, the filter gains can be achieved for the filtering problem under Case 2 as follows by solving the LMI in Theorem 4.1 with H_∞ performance $\gamma = 3.97$,

$$A_F = \begin{bmatrix} 0.9552 & -0.2565 \\ -0.0770 & 0.4281 \end{bmatrix}, B_F = \begin{bmatrix} -1.2769 \\ -2.6817 \end{bmatrix},$$
$$L_F = [\,-0.0296 \; -0.3258\,]$$

To further show the effectiveness of the obtained filter, let exogenous disturbance input $\omega(k) = 0.07\exp^{-0.3k}\cos(0.1\pi k)$ and the initial condition be $\phi(k) = [0.1 - 0.2]^T, k \in [-5, 0]$.

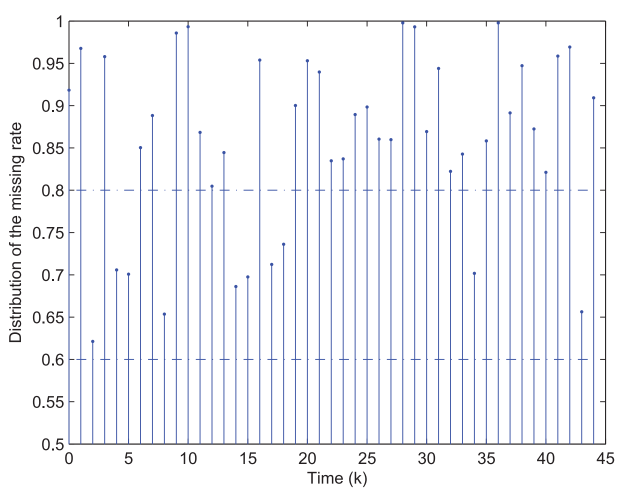

Figure 3. The distribution of the missing rate.

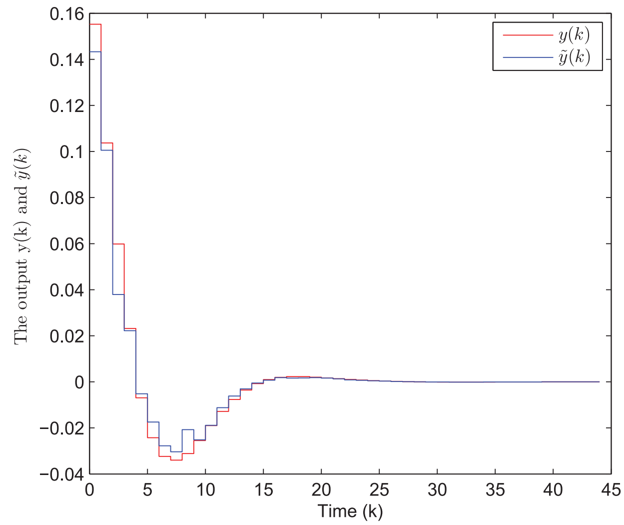

Figure 4. The output of $y(k)$ and $\tilde{y}(k)$.

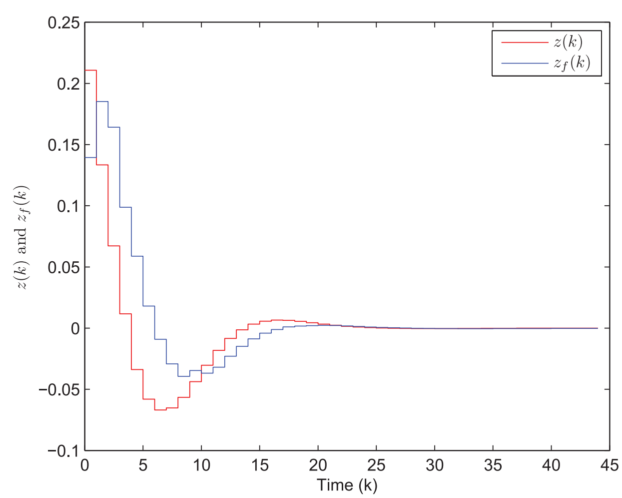

Figure 5. The output of $z(k)$ and $z_f(k)$.

Figure 3 shows the distribution of the missing rate, from which we can see that there exists a significant difference of the missing rate distribution between the interval [0.6, 0.8] and [0.8, 1.0]. It can be seen from Figure 4, under this stochastic missing rate, the measured output deviates its real signal by different degrees in every sampled time.

Figure 5 depicts the estimated results for the system (2). The simulation has confirmed that the designed filter performs very well even the system is subject to stochastic missing measurements.

6. Conclusion

In this paper, a filtering problem has been investigated for a linear delayed systems subject to stochastic missing measurements. A new stochastic missing measurement model is developed based on decomposing the varying interval of the missing rate into several intervals into which the probabilities of the practical measurement cast are different. We have established both the existence conditions and the explicit expression of the desired filters subject to stochastic missing measurements. A simulation example is presented to demonstrate the validity and less conservatism of the proposed approach.

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